Black Holes Without Singularities

A Conceptual Reinterpretation of Infinity in Gravitation

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Abstract

In classical general relativity, gravitational collapse under broad conditions leads to space-time singularities where curvature invariants formally diverge and the theory ceases to be predictive. These divergences are usually interpreted not as physical infinities, but as indications that the continuum description of space-time has reached its domain of validity. At the same time, several approaches to quantum gravity – such as string-theoretic fuzzball models, loop quantum gravity and Planck stars – suggest that the would-be singularity is replaced by a finite, highly quantum state.

This paper proposes a conceptual reinterpretation of black holes that makes this idea explicit: instead of treating singularities as "points of infinite density", we interpret the black hole itself as a magnitude-state of the collapsed mass–energy. In this picture, "infinity" is not a value attained by physical observables, but a domain marker signalling a transition from a classical configuration space to a new state space of black holes. A simple toy framework is introduced in which classical divergences are replaced by a mapping into a finite magnitude-state operator, capturing the total mass–energy while avoiding singular behaviour.

The goal is not to provide a full quantum gravity theory, but to offer a mathematically transparent and physically motivated interpretation of how one can treat ∞ in gravitation in an elegant and controlled way. This may serve as a conceptual bridge between classical singularity theorems and modern ideas about black-hole microphysics and information storage.

1 Introduction

Black holes are among the most robust predictions of Einstein's theory of general relativity. The Penrose–Hawking singularity theorems show that, under broad and physically reasonable conditions on the matter content, continued gravitational collapse inevitably leads to the formation of space-time singularities hidden behind event horizons. These singularities are characterised by geodesic incompleteness and the divergence of curvature invariants.

Physically, however, it is not clear how to interpret such divergences. It is widely accepted that they signal the breakdown of the classical continuum description rather than the existence of actual "infinite quantities" in nature. In parallel, microscopic models of black holes in candidate quantum gravity theories indicate that the singularity may be replaced by a finite, highly quantum state with a large but finite number of underlying degrees of freedom.

In this work, we develop a conceptual framework in which black holes are understood as magnitude-states of collapsed mass–energy. The key idea is that the appearance of ∞ in classical expressions – for instance, in curvature scalars or effective densities – should be treated as a domain marker: it indicates that we are attempting to extrapolate beyond the region where the classical observables are defined. At that boundary, the description is replaced by a new state, which we interpret as the black hole itself.

We present this idea in a deliberately simple form. First, we briefly review the classical picture of black holes and singularities in general relativity. Second, we survey existing proposals for singularity resolution. Third, we introduce a toy mathematical framework in which divergences are systematically traded for a magnitude-state operator. Finally, we discuss how this picture interfaces with questions of information, entropy and observer dependence.

2 Classical black holes and singularities in general relativity

In the simplest case, the Schwarzschild solution describes the exterior gravitational field of a static, spherically symmetric, uncharged mass. When matter collapses within its Schwarzschild radius, an event horizon forms: a null surface from which light and causal signals cannot reach asymptotic observers. Inside this horizon, the Schwarzschild metric predicts that all future-directed timelike geodesics end at $r = 0$ in finite proper time.

At $r = 0$, curvature invariants such as the Kretschmann scalar diverge. More importantly, from a geometric point of view, the space-time is geodesically incomplete: there exist inextendible geodesics of finite affine parameter length. The Penrose singularity theorem and its refinements generalise this situation: whenever matter satisfies suitable energy conditions and a trapped surface forms, geodesic incompleteness is inevitable.

It is crucial to emphasise that a singularity in this sense is not part of the manifold. It is not a point in space where "infinite density" is located; rather, it is a boundary of the space-time where the classical description simply stops. As such, the divergence of curvature is a symptom of the breakdown of the theory, not necessarily a property of the underlying physical reality.

This observation motivates the search for a description in which the end state of gravitational collapse is a well-defined, finite object. In the classical theory, this end state is encoded globally through the event horizon and asymptotic charges such as total mass, angular momentum and charge. In quantum gravity, one expects this end state to have a finite, albeit perhaps enormous, number of microstates.

3 Existing approaches to singularity resolution

Several research programmes in quantum gravity seek to replace the classical singularity by a finite, quantum state. Let us briefly recall three classes of ideas.

First, in string theory, the fuzzball proposal suggests that black holes are ensembles of horizon-sized string and brane configurations whose geometries differ from the classical black-hole solution at scales comparable to the horizon radius. In this picture, the region that would classically be occupied by a smooth geometry with a singular interior is instead filled by complicated but finite stringy structures. The singularity is absent; the would-be interior is replaced by a "fuzzball" of microstates.

Second, in loop quantum gravity, several models indicate that quantum gravitational effects can halt the collapse at high but finite densities and cause a bounce. In the Planck star scenario, for example, a collapsing star reaches a phase in which quantum gravity pressure balances gravity, forming a dense core that may eventually transition into a white hole as seen by external observers. Again, the classical singularity is replaced by a finite, quantum state.

Third, the holographic principle and black-hole thermodynamics point towards an interpretation where the relevant degrees of freedom of a black hole reside effectively on a lower-dimensional boundary, typically associated with the event horizon. Entropy scales with area rather than volume, suggesting that the information content is finite and proportional to the horizon area in Planck units.

These approaches differ strongly in their details and technical frameworks. Yet they share a common theme: where the classical theory predicts "infinite" curvature or density, the fundamental theory is taken to provide a finite, albeit extreme, state of quantum geometry. This motivates a more abstract, theory-agnostic way of talking about what replaces the singularity.

4 Infinity as a domain marker

The appearance of infinity in physics is almost always a sign that we are using a model outside its proper domain of applicability. In quantum field theory, ultraviolet divergences in loop integrals indicate that naïve continuum models break down at high energies or short distances and must be replaced by effective field theories or renormalised frameworks. Similarly, in gravitation, the divergence of curvature invariants signals that the classical continuum description of space-time is being extrapolated beyond where it should be trusted.

Rather than treating ∞ as a value that physical observables can truly assume, we therefore propose to interpret it as a domain marker. Concretely, suppose $Q(x)$ is a classical observable (such as a curvature scalar or energy density) defined on a region of space-time parametrised by x. If, according to the classical equations, $Q(x)$ diverges as x approaches some critical value x^*, we interpret this not as $Q(x^*) = \infty$, but as:

(i) $Q(x)$ is only defined classically on a domain D_classical, and
(ii) the limit $x \to x^*$ lies outside D_classical.

At x^*, one should transition to a different description, characterised by a different state space and different effective variables. In the context of black-hole formation, this transition is captured by the idea that the collapsing configuration is mapped into a new black-hole state that encodes the mass–energy and other conserved charges.

This point of view suggests introducing an explicit mapping from the classical configuration space into a new space of black-hole states, triggered whenever a classical quantity would formally diverge. In the next section we build a simple mathematical framework that makes this transition explicit.

5 A toy mathematical framework for handling divergences

We now introduce a toy mathematical framework that formalises the idea sketched above. The goal is not to reproduce the full complexity of general relativity, but to capture, in a simple setting, how divergences associated with gravitational collapse can be replaced by a mapping into finite magnitude-states.

We begin with a standard classical model for the density of a collapsing object, then impose a finite-density cutoff and define a transition condition. Finally, we introduce an abstract magnitude-state operator that encapsulates the collapsed mass–energy in a black-hole state, and we discuss how this can be viewed as an analogue of "division by zero" that avoids literal infinities.

5.1 Classical density blow-up

Consider a spherically symmetric collapsing body of total mass M and radius R. In a crude homogeneous model, the average mass density ρ(R) is given by

$$\rho(R) = M / V(R) = M / (4/3 \cdot \pi \cdot R^3) = 3M / (4\pi R^3).$$

As R decreases during collapse, ρ(R) increases. In the classical picture, nothing prevents R from tending to zero, so that ρ(R) → ∞ as R → 0. This divergence is associated with the formation of a singularity at the centre of the black hole.

However, we expect that at sufficiently high densities, quantum gravitational effects become important and the classical expression for ρ(R) ceases to be valid. Let us denote by ρ_max a critical density beyond which the classical continuum approximation breaks down. We do not need to know the precise microscopic origin of ρ_max; it merely acts as a proxy for the onset of new physics.

We can then define a critical radius R* at which the homogeneous density model would reach ρ_max:

$$\rho\_max = 3M / (4\pi R^{*3}) \Rightarrow R^* = (3M / (4\pi \rho\_max))^{(1/3)}.$$

For R > R*, the classical description is at least qualitatively meaningful. For R ≤ R*, the homogeneous continuum model attempts to push ρ beyond ρ_max and should no longer be trusted.

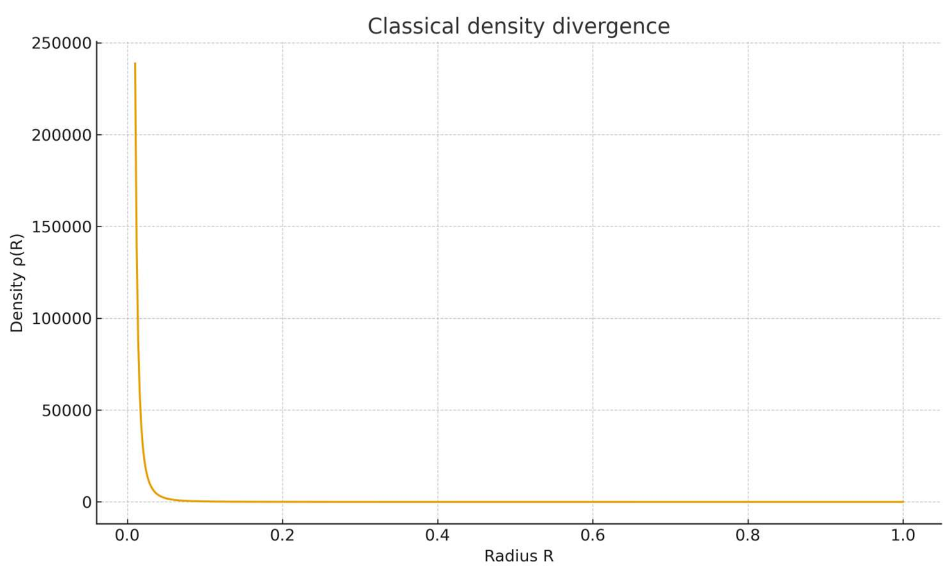

Figure 1: Classical density divergence in a homogeneous collapse model.
The function $\rho(R) = 3M/(4\pi R^3)$grows without bound as $R \to 0$, illustrating the classical singularity of the density term.

5.2 Saturated observables and transition condition

To avoid literal divergences, we define an effective, saturated density
ρ_eff(R) by ρ_eff(R) = min(ρ(R), ρ_max).

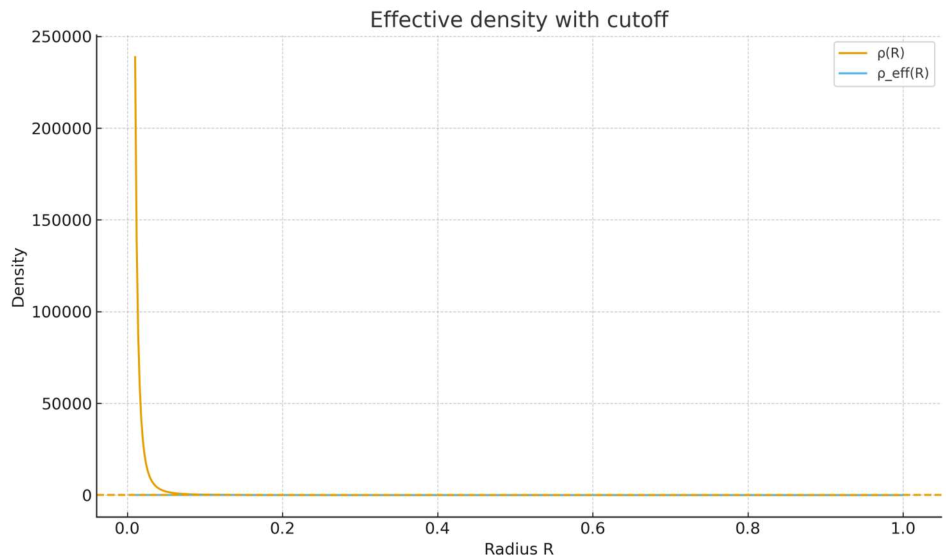

Figure 2: Effective density $\rho_{\text{eff}}(R)$with cutoff ρ_{max}.
The classical divergence is replaced by a saturation plateau at the critical radius R^*, marking the transition into the black-hole state regime.

This simple prescription ensures that ρ_eff(R) never exceeds ρ_max, even if the classical expression ρ(R) would diverge. However, the point of the construction is not merely to cap the density, but to signal a change in the underlying description when the saturation regime is entered.

We therefore associate to the collapsing configuration a pair (R, state) ,

where "state" tracks whether the system is still described by the classical continuum model or has transitioned into a black-hole magnitude-state. We define

state(R) = CLASSICAL for R > R*,
state(R) = BH_STATE for R ≤ R*.

In other words, the condition ρ(R) ≥ ρ_max – or equivalently R ≤ R* – is interpreted as the trigger for entering the black-hole regime. Once in BH_STATE, the internal dynamics are no longer described by the simple homogeneous density; instead, the system is characterised by macroscopic parameters such as its total mass M and possibly its angular momentum and charge.

5.3 Magnitude-state operator and a zero-division analogue

We now formalise the idea that the black hole is a magnitude-state of the collapsed mass–energy. Let C denote the space of classical configurations of collapsing matter (e.g. functions specifying density profiles and velocities), and let S_BH denote the space of black-hole states, characterised at least by a mass parameter M and possibly by additional charges.

We define a magnitude-state operator

B : C → S_BH,

which maps a classical configuration c ∈ C into a black-hole state B(c) once the critical condition ρ ≥ ρ_max is met somewhere in the configuration. In the homogeneous toy model, we can write B as a function of M alone:

B(M) ∈ S_BH.

Intuitively, B(M) represents "the black hole formed from collapsing mass M". The detailed internal structure of B(M) is left unspecified; it may encode, for example, a large number of microstates in a fundamental quantum theory.

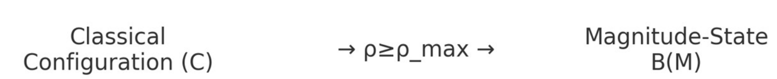

Figure 3: Conceptual mapping from classical configurations $C$to the magnitude-state $B(M)$, representing the black-hole state space S_{BH}.
The divergence of classical observables triggers the transition $C \rightarrow S_{\mathrm{BH}}$.

To connect this construction to the idea of "division by zero", consider again the homogeneous density

$$\rho(R) = 3M / (4\pi R^3).$$

Formally, the divergence $\rho(R) \to \infty$ as $R \to 0$ resembles the divergence of a function of the form N / V where the denominator V tends to zero. In naïve algebra, one might be tempted to treat this limit as a kind of division by zero. Instead of introducing ∞ as a numerical value, we propose to understand the limit

$$\lim_{R\to 0} 3M / (4\pi R^3)$$

as a signal that the classical observable ρ is leaving its domain of definition, and that the correct description is given by the magnitude-state B(M).

Symbolically, we can write

$$\lim_{R\to 0} 3M / (4\pi R^3) \rightsquigarrow B(M).$$

If one wishes to make contact with the intuition of "division by zero", this can be viewed as introducing a special operation ·/0 on positive reals M > 0 such that

$$M \cdot/0 := B(M),$$

where ·/0 is not ordinary division but a shorthand for "collapse into a black-hole state". Crucially, this operation returns an element of S_BH rather than a real number; it is therefore not subject to the usual algebraic rules of real arithmetic and avoids the contradictions that arise when one tries to define 1/0 as a real number.

In this sense, the "mathematical" analogue of dividing by zero is replaced by a controlled transition from the classical configuration space C to the black-hole state space S_BH via the operator B.

6 Application to gravitational collapse

Let us summarise how the above framework applies to the process of gravitational collapse in a simplified setting. Although we work with a toy model, the structure mirrors qualitative expectations from more sophisticated treatments.

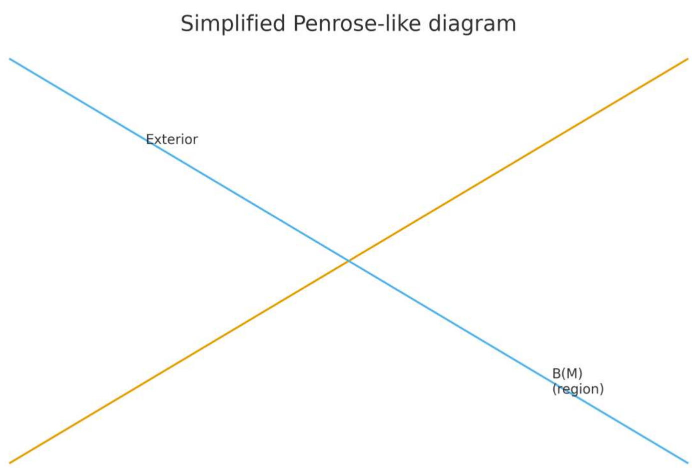

Figure 4: Simplified Penrose-like schematic illustrating the replacement of the classical singularity by the magnitude-state region $B(M)$.
The exterior region and horizon structure remain classical, while the interior is reinterpreted as a finite state domain.

6.1 Collapse scenario in the toy model

1. Start from an extended classical configuration $c_0 \in C$, representing a star of mass M and radius $R_0 \gg R^*$.

2. As the star collapses due to its own gravity, its radius R(t) decreases and its average density ρ(R(t)) increases.

3. As long as $\rho(R(t)) < \rho_{max}$, the system is in the CLASSICAL state. Its dynamics are well described (at least qualitatively) by general relativity coupled to suitable matter models.

4. When R(t) reaches R*, the homogeneous density model would predict $\rho(R^*) = \rho_{max}$. For smaller radii, the classical expression attempts to push ρ beyond ρ_{max}. Instead of accepting $\rho \to \infty$, we interpret this as the point where the mapping

$$c(t) \to B(M)$$

becomes the relevant description. The system transitions into the black-hole state B(M), and the internal classical degrees of freedom are no longer treated explicitly.

5. For all subsequent times, the exterior geometry is described by an appropriate black-hole space-time (e.g. a member of the Kerr–Newman family), and the interior is represented abstractly by B(M). Microscopic models such as fuzzballs or Planck stars provide candidate realisations of B(M) in specific quantum gravity frameworks.

In this narrative, there is no stage at which a physical quantity literally becomes infinite. Instead, the attempt of the classical model to drive ρ beyond ρ_{max} is recognised as an extrapolation beyond its domain of validity, and the description is replaced by a finite magnitude-state.

7 Discussion: information, entropy, and observers

The magnitude-state interpretation has several conceptual consequences for how we think about information, entropy and observers in black-hole space-times.

From the perspective of an external observer, the black hole is characterised by a small set of macroscopic parameters. The magnitude-state B(M) encodes, abstractly, the vast number of microscopic configurations consistent with these parameters. Black-hole entropy can then be viewed as a measure of the logarithm of the number of such microstates.

Importantly, in this picture no fundamental information is destroyed at a singularity, because there is no singularity as a physical object. Rather, information is hidden in B(M) and possibly on or near the horizon, in line with holographic ideas. The information paradox is not solved by this framework, but it is at least reformulated: the central question becomes how the information encoded in B(M) is released or transformed during black-hole evaporation, rather than whether it is lost at an infinitely dense point.

Observer dependence also becomes more transparent. For an infalling observer crossing the horizon, the transition to B(M) may be experienced very differently from the perspective of a distant observer. The present framework deliberately focuses on the description accessible to asymptotic observers, leaving the detailed interior experience to be specified by a more complete microscopic theory.

Finally, by treating ∞ as a domain marker rather than a value, we avoid attributing unphysical properties to nature. The role of the singularity theorems is then to tell us when such domain boundaries are inevitable within classical general relativity, not to assert that the universe contains actual infinities.

8 Conclusion and outlook

We have proposed a conceptual reinterpretation of black holes that avoids treating singularities as physical objects with infinite density or curvature. Instead, black holes are regarded as magnitude-states B(M) of collapsed mass–energy. Classical divergences, such as the blow-up of homogeneous density as $R \to 0$, are understood as indicators that one has reached the boundary of the classical description's domain of validity. At that point, the configuration is mapped into a finite black-hole state.

To make this idea concrete, we introduced a simple toy framework in which a critical density ρ_{max} defines a transition radius R^*. For $R > R^*$, a classical continuum model applies; for $R \leq R^*$, the system is described by a black-hole magnitude-state B(M). Symbolically, the would-be divergence of quantities such as $3M/(4\pi R^3)$ does not produce literal infinities, but triggers the mapping into S_{BH}.

This framework is deliberately modest. It does not supply a specific microphysical model for B(M) and does not attempt to calculate black-hole entropy, Hawking radiation spectra or evaporation times. Nevertheless, it offers a clean language in which to talk about the role of infinity in gravitational collapse.

Several extensions suggest themselves. One could refine the toy model to include non-homogeneous collapse, angular momentum, charge, and dynamical horizons. One could also attempt to embed the magnitude-state idea into concrete quantum-gravity approaches, identifying B(M) with specific objects such as fuzzballs, Planck stars or holographic boundary states. Finally, it would be interesting to explore whether similar magnitude-state transitions can clarify the status of cosmological singularities, such as the big bang, in a parallel fashion.

In this sense, the present work should be seen as a conceptual bridge: it connects rigorous singularity theorems and quantum-gravity motivated singularity resolution scenarios by providing an explicit, mathematically controlled way to replace ∞ with a finite, information-carrying state.

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